# Exercise 33

(a) If $f(x) = x^4 + 2x$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of $f$ and $f'$.

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**Solution**

Calculate the derivative of $f(x)$ using the definition.

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\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[(x+h)^4 + 2(x+h)] - [x^4 + 2x]}{h} \\
&= \lim_{h\to 0} \frac{[(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) + 2x + 2h] - x^4 - 2x}{h} \\
&= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 2h}{h} \\
&= \lim_{h\to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3 + 2)}{h} \\
&= \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3 + 2) \\
&= 4x^3 + 2
\end{aligned}
$$

Below is a graph of $f(x)$ and $f'(x)$ versus $x$.